

Stabilization of 2-D Markovian Jump Systems in Roesser Model

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Abstract

This paper is concerned with the problem of stabilizing controller design for 2-D systems with Markovian jump parameters. The mathematical model of 2-D jump systems is established upon the well-known Roesser model, and sufficient conditions are obtained for the existence of desired controllers in terms of linear matrix inequalities (LMIs), which can be readily solved by available numerical software. A numerical example is provided to show the applicability of the proposed theories.

1 Introduction

Two-dimensional (2-D) model represents a wide range of practical systems, such as those in image data processing and transmission, thermal processes, gas absorption and water stream heating etc. [9, 11]. Therefore, in recent years 2-D discrete systems received much attention, and many important results are easily available in the literature. To mention a few, [8, 10] investigate the stability of 2-D systems, [5, 15] are concerned with the controller and filter design problems, and [6] addresses the model approximation problem for 2-D digital filters etc.

On the other hand, it has been well recognized that many practical systems are subject to abrupt changes. A very popular way to characterize jump linear systems is the so-called Markovian switching modeling, which assumes that the system under investigation operates at several modes, and the mode switching is governed by a Markov process. Many important results have been reported for this kind of systems, see, for instance, the control problems are solved in [2, 3, 13], the filtering problems are investigated in [4, 14], and the model reduction problem has also been reported in [16]. However, the aforementioned results are only concerned with one-dimensional (1-D) systems, and to the best of the authors' knowledge, no effort has been made toward investigating the problems arising in 2-D jump systems.

In this paper, we further extend the results obtained for 1-D jump systems, to investigate the problem of stochastic stabilization of 2-D systems with

Markovian jump parameters. The mathematical model of 2-D jump systems is established upon the well-known Roesser model, and sufficient conditions are obtained for the existence of desired controllers in terms of linear matrix inequalities (LMIs), which can be readily solved by available numerical software.

The remainder of this paper is organized as follows. The problem of stochastic stabilization of 2-D Markovian jump systems is formulated in Section 2. The main result is presented in Section 3. Section 4 provides an illustrative example and we conclude this paper in Section 5.

Notations: The notations used throughout the paper are fairly standard. The superscript " T " stands for matrix transposition; \mathbb{R}^n denotes the n -dimensional Euclidean space, $\mathbb{R}^{m \times n}$ is the set of all real matrices of dimension $m \times n$ and the notation $P > 0$ means that P is real symmetric and positive definite; I and 0 represent identity matrix and zero matrix; $\|\cdot\|$ refers to the Euclidean vector norm; and $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of a real symmetric matrix respectively. In symmetric block matrices or long matrix expressions, we use an asterisk (*) to represent a term that is induced by symmetry and $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. In addition, $E\{x\}$ and $E\{x | y\}$ will, respectively, mean expectation of x and expectation of x conditional on y .

2 Problem Formulation

Consider the following 2-D discrete system in Roesser model with Markovian jump parameters:

$$S: \begin{bmatrix} x_{i,j+1}^h \\ x_{i,j+1}^v \end{bmatrix} = A(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + B(r_{i,j})u_{i,j} \quad (1)$$

where $x_{i,j}^h \in \mathbb{R}^{n_1}$, $x_{i,j}^v \in \mathbb{R}^{n_2}$ represent the horizontal and vertical states, respectively; $u_{i,j} \in \mathbb{R}^k$ is the control input. $A(r_{i,j})$, $B(r_{i,j})$ are appropriately dimensioned real valued system matrices. These matrices are functions of $r_{i,j}$, which takes values in a finite set $\mathcal{L} = \{1, \dots, S\}$, with transition probabilities

$$\Pr\{r_{i+1,j} = n | r_{i,j} = m\}$$

$$= \Pr \{r_{i,j+1} = n \mid r_{i,j} = m\} = p_{mn}$$

Here $p_{mn} \geq 0$ and, for any $m \in \mathcal{L}$,

$$\sum_{n=1}^S p_{mn} = 1$$

Remark 1 For simplicity, here the probabilities of jump in each direction are assumed to be the same. However, the theories developed in this note can be easily adapted to cope with systems with different transition probabilities along different directions.

In the case there is only one mode, that is, the matrices $A(r_{i,j})$, $B(r_{i,j})$ take fixed values A , B , system S becomes the standard 2-D discrete system in Roesser model as follows [12]:

$$\mathcal{R} : \begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = A \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + Bu_{i,j} \quad (2)$$

To simplify the notation, when the system operates at the m th mode, that is, $r_{i,j} = m$, the matrices $A(r_{i,j})$, $B(r_{i,j})$ are denoted as A_m , B_m respectively. Unless otherwise stated, similar simplification is also applied to other matrices in the following.

Throughout the paper, we denote the system state as $x_{i,j} = [x_{i,j}^h \ x_{i,j}^v]^T$. The boundary condition (X_0, R_0) are defined as follows:

$$\begin{aligned} X_0 &= [x_{0,0}^{hT} \ x_{0,1}^{hT} \ x_{0,2}^{hT} \ \cdots \ x_{0,0}^{vT} \ x_{1,0}^{vT} \ x_{2,0}^{vT} \ \cdots]^T \\ R_0 &= \{r_{0,0}, r_{0,1}, r_{0,2}, \dots, r_{0,0}, r_{1,0}, r_{2,0}, \dots\} \end{aligned}$$

Then, we make the following assumption on the boundary condition.

Assumption 1: The boundary condition is assumed to satisfy

$$\lim_{N \rightarrow \infty} \mathbb{E} \left\{ \sum_{k=0}^N (|x_{0,k}^h|^2 + |x_{k,0}^v|^2) \right\} < \infty$$

We first introduce the following definition, which will be essential for our derivation.

Definition 1 The 2-D jump system S in (1) with $u_{i,j} \equiv 0$ is said to be mean-square asymptotically stable if

$$\lim_{i+j \rightarrow \infty} \mathbb{E} \{ |x_{i,j}|^2 \} = 0$$

for every boundary condition (X_0, R_0) satisfying Assumption 1.

In this note, it is assumed that perfect observation of $r_{i,j}$ and $x_{i,j}$ are available at each (i, j) , upon which we will deal with the problems of stochastic stabilization for the 2-D jump system S in (1). More specifically, for the problem of stochastic stabilization, we are concerned with the design of a state feedback control law

$$u_{i,j} = K(r_{i,j})x_{i,j} \quad (3)$$

where $K(r_{i,j}) = K_m$ when $r_{i,j} = m$ (constant for each value of m), such that the resulting closed-loop system is mean-square asymptotically stable. In this case, system S is said to be stochastically stabilizable.

3 Stochastic Stabilization

Theorem 1 The 2-D jump system S in (1) is stochastically stabilizable if there exist matrices $Y_m^h > 0$, $Y_m^v > 0$, \bar{K}_m , $m = 1, \dots, S$, satisfying

$$\begin{bmatrix} -Y_m & \Psi_1 \\ * & \Psi_2 \end{bmatrix} < 0, \quad m = 1, \dots, S \quad (4)$$

where $Y_m = \text{diag} \{Y_m^h, Y_m^v\}$, $m = 1, \dots, S$, and

$$\begin{aligned} \Psi_1 &= [Y_m A_m^T + \bar{K}_m^T B_{1m}^T \ \cdots \ Y_m A_m^T + \bar{K}_m^T B_{1m}^T] \\ \Psi_2 &= \text{diag} \{-p_{m1}^{-1} Y_1, \dots, -p_{mS}^{-1} Y_S\} \end{aligned}$$

In this case, the gain matrix of an admissible control law in the form of (3) is given by

$$K_m = \bar{K}_m Y_m^{-1} \quad (5)$$

Proof: Applying the control law in (3) to system S in (1), we obtain the following closed-loop system

$$C_1 : \begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \bar{A}(r_{i,j}) \begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix}$$

where

$$\bar{A}(r_{i,j}) = A(r_{i,j}) + B_1(r_{i,j})K(r_{i,j}) \quad (6)$$

Define $P_m \triangleq \text{diag} \{P_m^h, P_m^v\} = Y_m^{-1}$, $m = 1, \dots, S$, then by performing a congruence transformation to (4) by $\text{diag} \{Y_m^{-1}, I\}$ and by considering (5) and (6), (4) is equivalent to

$$\begin{bmatrix} -P_m & \Phi_1 \\ * & \Phi_2 \end{bmatrix} < 0, \quad m = 1, \dots, S \quad (7)$$

where

$$\begin{aligned} \Phi_1 &= [\bar{A}_m^T \ \cdots \ \bar{A}_m^T], \\ \Phi_2 &= \text{diag} \{-p_{m1}^{-1} P_1^{-1}, \dots, -p_{mS}^{-1} P_S^{-1}\} \end{aligned} \quad (8)$$

By Schur Complement [1], (7) is equivalent to

$$\Upsilon_m \triangleq \bar{A}_m^T \bar{P}_m \bar{A}_m - P_m < 0, \quad m = 1, \dots, S \quad (9)$$

where $\bar{P}_m = \sum_{n=1}^S p_{mn} P_n$.

Now consider the following index

$$\mathcal{I}_{i,j} \triangleq \mathbb{E} \left\{ \left[\begin{array}{c} x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h \\ + x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v \\ - x_{i,j}^T P(r_{i,j}) x_{i,j} \end{array} \right] \middle| (x_{i,j}, r_{i,j} = m) \right\} \quad (10)$$

where $P(r_{i,j}) = \text{diag}\{P^h(r_{i,j}), P^v(r_{i,j})\}$, which is denoted as P_m when $r_{i,j} = m$. Note that P_m is constant for each m . Then along the solution of the closed-loop system \mathcal{C}_1 in (6), we have

$$\begin{aligned} & \mathcal{I}_{i,j} \\ &= \sum_{n=1}^S \left\{ \begin{aligned} & \Pr(r_{i+1,j} = n | r_{i,j} = m) x_{i+1,j}^{hT} P_n^h x_{i+1,j}^h \\ & + \Pr(r_{i,j+1} = n | r_{i,j} = m) x_{i,j+1}^{vT} P_n^v x_{i,j+1}^v \\ & - x_{i,j}^T P_m x_{i,j} \end{aligned} \right\} \\ &= x_{i,j}^T \Upsilon_m x_{i,j} \end{aligned}$$

This means that for all $x_{i,j} \neq 0$, we have

$$\begin{aligned} & \mathbb{E} \left\{ \frac{\left[\begin{array}{c} x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h \\ + x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v \\ - x_{i,j}^T P(r_{i,j}) x_{i,j} \end{array} \right]}{x_{i,j}^T P(r_{i,j}) x_{i,j}} \middle| (x_{i,j}, r_{i,j}) \right\} \\ &= - \frac{x_{i,j}^T [-\Upsilon(r_{i,j})] x_{i,j}}{x_{i,j}^T P(r_{i,j}) x_{i,j}} \leq - \min_{m \in \mathcal{L}} \frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)} \\ &= \alpha - 1 \end{aligned}$$

where $\alpha \triangleq 1 - \min_{m \in \mathcal{L}} \frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)}$. Since $\min_{m \in \mathcal{L}} \frac{\lambda_{\min}(-\Upsilon_m)}{\lambda_{\max}(P_m)} > 0$, we have $\alpha < 1$. Obviously,

$$\alpha \geq \frac{\mathbb{E} \left\{ \frac{\left[\begin{array}{c} x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h \\ + x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v \end{array} \right]}{x_{i,j}^T P(r_{i,j}) x_{i,j}} \middle| (x_{i,j}, r_{i,j}) \right\}}{1} > 0$$

That is α belongs to $(0, 1)$ and is independent of $x_{i,j}$. Therefore, we have

$$\begin{aligned} & \mathbb{E} \left\{ \frac{\left[\begin{array}{c} x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h \\ + x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v \end{array} \right]}{x_{i,j}^T P(r_{i,j}) x_{i,j}} \middle| (x_{i,j}, r_{i,j}) \right\} \\ &\leq \alpha x_{i,j}^T P(r_{i,j}) x_{i,j} \end{aligned}$$

Taking expectation of both sides yields

$$\mathbb{E} \left\{ \frac{x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h}{+ x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v} \right\} \leq \alpha \mathbb{E} \{ x_{i,j}^T P(r_{i,j}) x_{i,j} \}$$

That is,

$$\begin{aligned} & \mathbb{E} \left\{ \frac{x_{i+1,j}^{hT} P^h(r_{i+1,j}) x_{i+1,j}^h}{+ x_{i,j+1}^{vT} P^v(r_{i,j+1}) x_{i,j+1}^v} \right\} \\ &\leq \alpha \mathbb{E} \{ x_{i,j}^T P^h(r_{i,j}) x_{i,j}^h + x_{i,j}^{vT} P^v(r_{i,j}) x_{i,j}^v \} \quad (11) \end{aligned}$$

Upon the relationship (11), it can be established that

$$\begin{aligned} & \mathbb{E} \{ x_{0,k+1}^{hT} P^h(r_{0,k+1}) x_{0,k+1}^h \} \\ &= \mathbb{E} \{ x_{0,k+1}^{hT} P^h(r_{0,k+1}) x_{0,k+1}^h \} \\ & \mathbb{E} \left\{ \frac{x_{1,k}^{hT} P^h(r_{1,k}) x_{1,k}^h}{+ x_{0,k+1}^{vT} P^v(r_{0,k+1}) x_{0,k+1}^v} \right\} \\ &\leq \alpha \mathbb{E} \left\{ \frac{x_{0,k}^{hT} P^h(r_{0,k}) x_{0,k}^h}{+ x_{0,k}^{vT} P^v(r_{0,k}) x_{0,k}^v} \right\} \end{aligned}$$

$$\begin{aligned} & \mathbb{E} \left\{ \frac{x_{2,k-1}^{hT} P^h(r_{2,k-1}) x_{2,k-1}^h}{+ x_{1,k}^{vT} P^v(r_{1,k}) x_{1,k}^v} \right\} \\ &\leq \alpha \mathbb{E} \left\{ \frac{x_{1,k-1}^{hT} P^h(r_{1,k-1}) x_{1,k-1}^h}{+ x_{1,k-1}^{vT} P^v(r_{1,k-1}) x_{1,k-1}^v} \right\} \\ &\vdots \\ & \mathbb{E} \left\{ \frac{x_{k+1,0}^{hT} P^h(r_{k+1,0}) x_{k+1,0}^h}{+ x_{k,1}^{vT} P^v(r_{k,1}) x_{k,1}^v} \right\} \\ &\leq \alpha \mathbb{E} \left\{ \frac{x_{k,0}^{hT} P^h(r_{k,0}) x_{k,0}^h}{+ x_{k,0}^{vT} P^v(r_{k,0}) x_{k,0}^v} \right\} \\ & \mathbb{E} \{ x_{k+1,0}^{vT} P^v(r_{k+1,0}) x_{k+1,0}^v \} \\ &= \mathbb{E} \{ x_{k+1,0}^{vT} P^v(r_{k+1,0}) x_{k+1,0}^v \} \end{aligned}$$

Adding both sides of the above inequality system yields

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{j=0}^{k+1} \left[\frac{x_{k+1-j,j}^{hT} P^h(r_{k+1-j,j}) x_{k+1-j,j}^h}{+ x_{k+1-j,j}^{vT} P^v(r_{k+1-j,j}) x_{k+1-j,j}^v} \right] \right\} \\ &\leq \alpha \mathbb{E} \left\{ \sum_{j=0}^k \left[\frac{x_{k-j,j}^{hT} P^h(r_{k-j,j}) x_{k-j,j}^h}{+ x_{k-j,j}^{vT} P^v(r_{k-j,j}) x_{k-j,j}^v} \right] \right\} \\ &+ \mathbb{E} \left\{ \frac{x_{0,k+1}^{hT} P^h(r_{0,k+1}) x_{0,k+1}^h}{+ x_{k+1,0}^{vT} P^v(r_{k+1,0}) x_{k+1,0}^v} \right\} \end{aligned}$$

Using this relationship iteratively, we obtain

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{j=0}^{k+1} \left[\frac{x_{k+1-j,j}^{hT} P^h(r_{k+1-j,j}) x_{k+1-j,j}^h}{+ x_{k+1-j,j}^{vT} P^v(r_{k+1-j,j}) x_{k+1-j,j}^v} \right] \right\} \\ &\leq \alpha^{k+1} \mathbb{E} \left\{ \frac{x_{0,0}^{hT} P^h(r_{0,0}) x_{0,0}^h}{+ x_{0,0}^{vT} P^v(r_{0,0}) x_{0,0}^v} \right\} \\ &+ \mathbb{E} \left\{ \sum_{j=0}^k \alpha^j \left[\frac{x_{0,k+1-j}^{hT} P^h(r_{0,k+1-j}) x_{0,k+1-j}^h}{+ x_{k+1-j,0}^{vT} P^v(r_{k+1-j,0}) x_{k+1-j,0}^v} \right] \right\} \\ &= \mathbb{E} \left\{ \sum_{j=0}^{k+1} \alpha^j \left[\frac{x_{0,k+1-j}^{hT} P^h(r_{0,k+1-j}) x_{0,k+1-j}^h}{+ x_{k+1-j,0}^{vT} P^v(r_{k+1-j,0}) x_{k+1-j,0}^v} \right] \right\} \end{aligned}$$

Therefore, we have

$$\begin{aligned} & \mathbb{E} \left\{ \sum_{j=0}^{k+1} \left[|x_{k+1-j,j}^h|^2 + |x_{k+1-j,j}^v|^2 \right] \right\} \quad (12) \\ &\leq \kappa \sum_{j=0}^{k+1} \alpha^j \mathbb{E} \left\{ |x_{0,k+1-j}^h|^2 + |x_{k+1-j,0}^v|^2 \right\} \end{aligned}$$

where

$$\kappa \triangleq \frac{\max_{m \in \mathcal{L}} \{\lambda_{\max}(P_m)\}}{\min_{m \in \mathcal{L}} \{\lambda_{\min}(P_m)\}}$$

$$\text{Now, denote } \mathcal{X}_k \triangleq \sum_{j=0}^k \left[|x_{k-j,j}^h|^2 + |x_{k-j,j}^v|^2 \right],$$

then upon the inequality (12) we have

$$\mathbb{E} \{ \mathcal{X}_0 \} \leq \kappa \mathbb{E} \{ |x_{0,0}^h|^2 + |x_{0,0}^v|^2 \}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{X}_1\} &\leq \kappa \begin{bmatrix} \alpha \mathbb{E}\{x_{0,0}^h|^2 + |x_{0,0}^v|^2\} \\ + \mathbb{E}\{x_{0,1}^h|^2 + |x_{1,0}^v|^2\} \end{bmatrix} \\
\mathbb{E}\{\mathcal{X}_2\} &\leq \kappa \begin{bmatrix} \alpha^2 \mathbb{E}\{x_{0,0}^h|^2 + |x_{0,0}^v|^2\} \\ + \alpha \mathbb{E}\{x_{0,1}^h|^2 + |x_{1,0}^v|^2\} \\ + \mathbb{E}\{x_{0,2}^h|^2 + |x_{2,0}^v|^2\} \end{bmatrix} \\
&\vdots \\
\mathbb{E}\{\mathcal{X}_N\} &\leq \kappa \begin{bmatrix} \alpha^N \mathbb{E}\{x_{0,0}^h|^2 + |x_{0,0}^v|^2\} \\ + \alpha^{N-1} \mathbb{E}\{x_{0,1}^h|^2 + |x_{1,0}^v|^2\} \\ + \cdots + \mathbb{E}\{x_{0,N}^h|^2 + |x_{N,0}^v|^2\} \end{bmatrix}
\end{aligned}$$

Adding both sides of the above inequality system yields

$$\begin{aligned}
&\sum_{k=0}^N \mathbb{E}\{\mathcal{X}_k\} \\
&\leq \kappa(1 + \alpha + \cdots + \alpha^N) \mathbb{E}\{x_{0,0}^h|^2 + |x_{0,0}^v|^2\} \\
&\quad + \kappa(1 + \alpha + \cdots + \alpha^{N-1}) \mathbb{E}\{x_{0,1}^h|^2 + |x_{1,0}^v|^2\} \\
&\quad + \cdots + \kappa \mathbb{E}\{x_{0,N}^h|^2 + |x_{N,0}^v|^2\} \\
&\leq \kappa(1 + \alpha + \cdots + \alpha^N) \mathbb{E}\{x_{0,0}^h|^2 + |x_{0,0}^v|^2\} \\
&\quad + \kappa(1 + \alpha + \cdots + \alpha^N) \times \mathbb{E}\{x_{0,1}^h|^2 + |x_{1,0}^v|^2\} \\
&\quad + \cdots + \kappa(1 + \alpha + \cdots + \alpha^N) \mathbb{E}\{x_{0,N}^h|^2 + |x_{N,0}^v|^2\} \\
&= \kappa \frac{1 - \alpha^N}{1 - \alpha} \mathbb{E}\left\{\sum_{k=0}^N [x_{0,k}^h|^2 + |x_{k,0}^v|^2]\right\}
\end{aligned}$$

Then, under Assumption 1, the right side of the above inequality is bounded, which means $\lim_{k \rightarrow \infty} \mathbb{E}\{\mathcal{X}_k\} = 0$, that is, $\mathbb{E}\{|x_{i,j}|^2\} \rightarrow 0$ as $i + j \rightarrow \infty$, then by Definition 1, the closed-loop system \mathcal{C}_1 in (6) is mean-square asymptotically stable. \square

Remark 2 Theorem 1 provides strict LMI conditions for the existence of admissible controllers, which can be readily solved using standard numerical software [7].

In the case that there is only one mode in system \mathcal{S} (that is, system \mathcal{S} degrades to the standard 2-D discrete system \mathcal{R} described by (2)), Theorem 1 is specialized as follows.

Corollary 2 The 2-D discrete system \mathcal{R} in (2) is stabilizable if there exist matrices $Y^h > 0$, $Y^v > 0$, \bar{K} satisfying

$$\begin{bmatrix} -Y & Y A^T + \bar{K}^T B_1^T \\ * & -Y \end{bmatrix} < 0 \quad (13)$$

where $Y = \text{diag}\{Y^h, Y^v\}$. In this case, an admissible control law is given by

$$u_{i,j} = K x_{i,j}, \quad K = \bar{K} Y^{-1} \quad (14)$$

4 Illustrative Example

Consider system \mathcal{S} in (1) involving two modes. For mode 1, the system matrices are given by

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$$

For mode 2, the system matrices are given by

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$$

Assume that the transition probability matrix is given by

$$p = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

Using the result developed above, the obtained feedback gains for a desired stabilizing controller are given by $K_1 = \begin{bmatrix} -0.0081 & -9.9986 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0.2795 & -10.0105 \end{bmatrix}$.

5 Concluding Remarks

The problems of stochastic stabilization of 2-D systems with Markovian jump parameters are investigated in this paper. Sufficient conditions are obtained for the existence of desired controllers in terms of LMIs. These obtained results can be further extended to more general cases whose system matrices also contain parameter uncertainties represented by either polytopic or norm-bounded approaches. A numerical example is provided to show the applicability of the proposed theories.

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